

Mathematics
Entry Exam 0
Length: 3 hours



Name and surname: _____

School's name: _____

Instructions to candidates

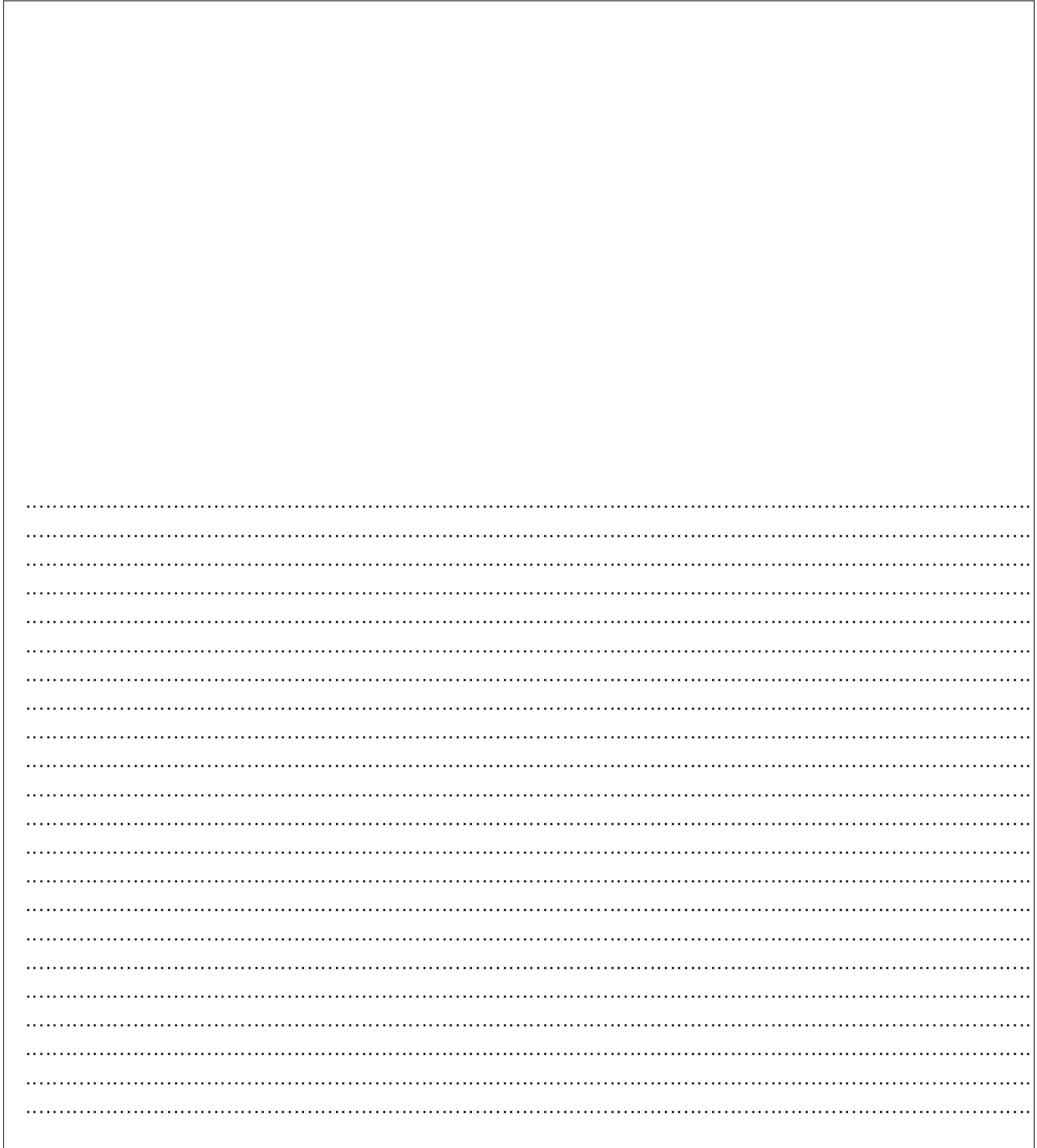
- Write your name and school affiliation above.
- Do not open the examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: Answer all questions in the boxes provided.
- The maximum mark for this examination paper is [75 marks].

Section A

1. [Maximum mark: 8]

a) Sketch on the same axes the curve $y = \left| \frac{2}{x-2} \right|$ and the line $y = x + 1$, clearly indicating any asymptotes and any axes intercepts.

b) Find the exact solutions to the equation $x + 1 = \left| \frac{2}{x-2} \right|$.

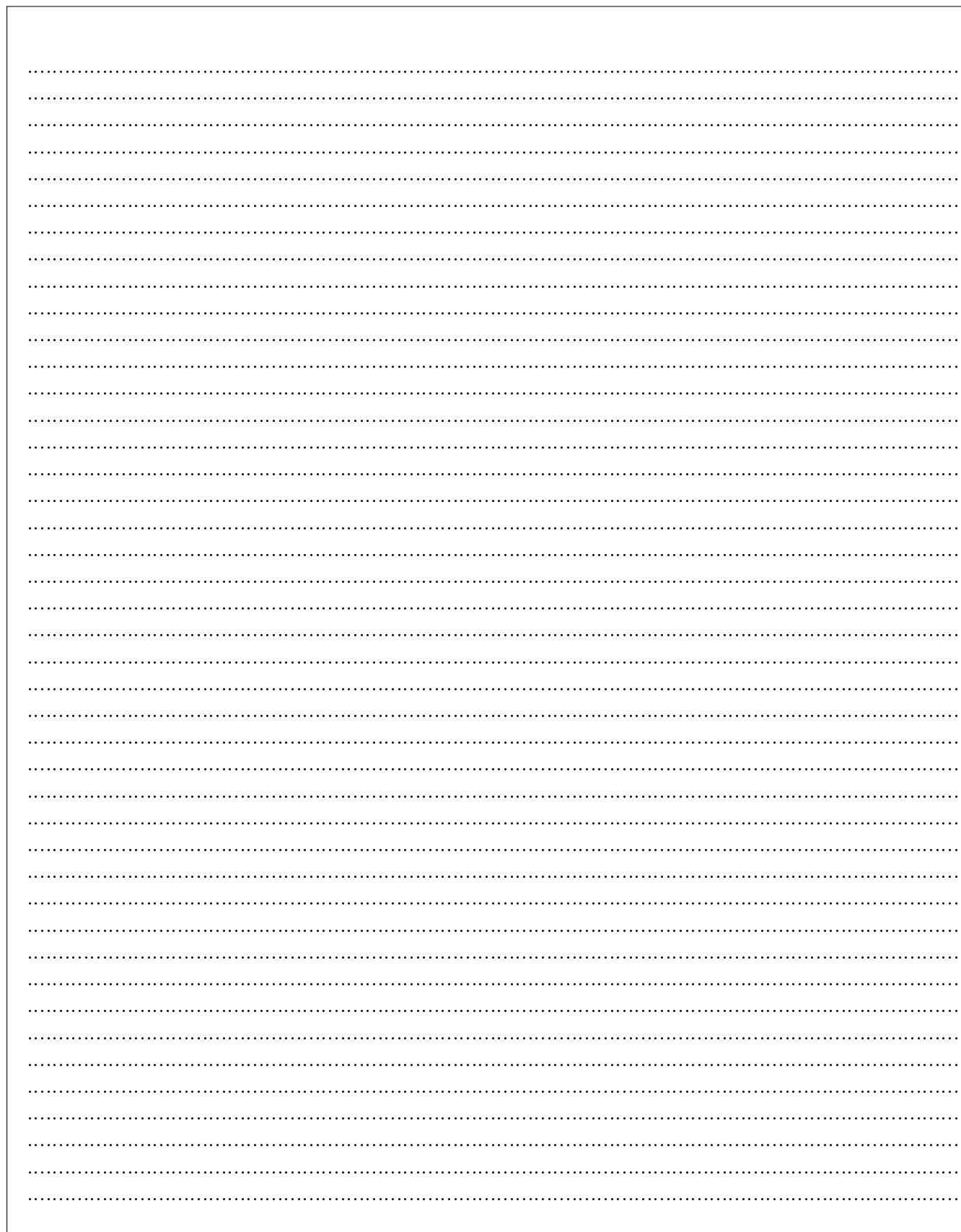


2. [Maximum mark: 12]

The numbers shown by two six-sided fair dice are labelled n_1 and n_2 .

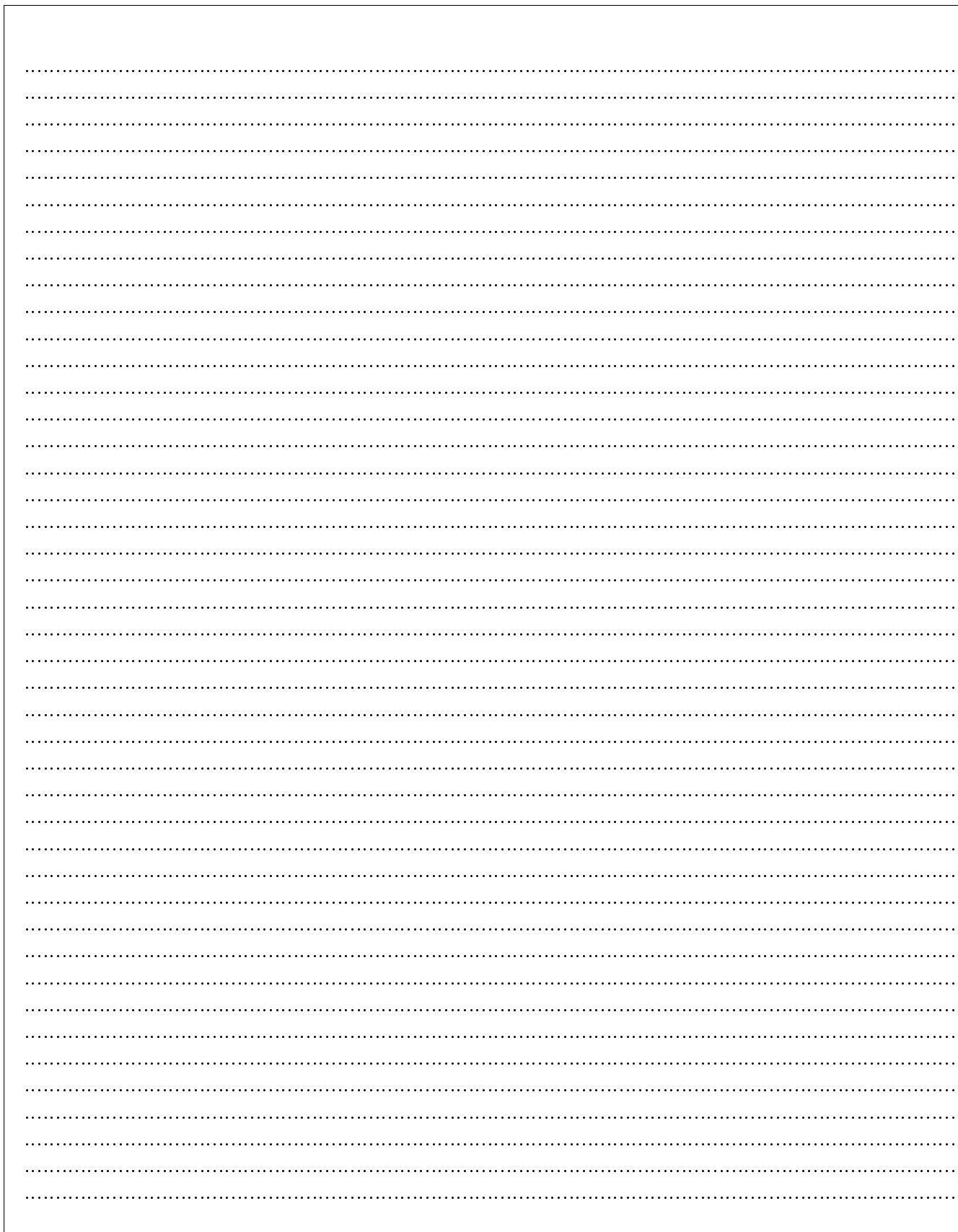
We thus consider the expression : $S = an_1 + bn_2 + c$ where $(a, b, c) \in \mathbb{R}^3$.

Explaining in detail your proof, find the values of a, b, c such as the range of possible values for S covers all integers from 0 to 35 following a uniform law



3. [Maximum mark: 15]

Show that, among 9 points in the rectangle $R = \{(x, y) \in \mathbb{R}^2 / -1 \leq x \leq 1, 0 \leq y \leq 1\}$ there is two points whose distance is less or equal to $\frac{1}{\sqrt{2}}$.



Section B

6. [Maximum mark: 22]

a) Use de Moivre's theorem to find the value of $(\cos(\frac{\pi}{5}) + i \sin(\frac{\pi}{5}))^5$.

b) Prove that for all $n \in \mathbb{Z}^+$,

$$(\cos \theta - i \sin \theta)^n = \cos n\theta - i \sin n\theta$$

Let $\omega = \cos(\frac{2\pi}{5}) + i \sin(\frac{2\pi}{5})$.

c) Show that $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$.

Let $z = \omega + \omega^{-1}$.

d) Find real numbers a, b, c such that $az^2 + bz + c = 0$.

e) Hence find the exact values of $\cos(2\pi/5)$, $\sin(2\pi/5)$ and $\tan(2\pi/5)$.

f) Find a right triangle with hypotenuse equal to $\sqrt{5}$ and other sides with integer lengths.

g) From the previous results, deduce a method for drawing a regular convex pentagon with a compass and a rule.